# <span id="page-0-0"></span>Open-set Domain Adaptation via Joint Error based Multi-class Positive and Unlabeled Learning ECCV 2024

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## Problem Setting



Figure: Given labeled data from source domain  $S': \mathcal{X} \times (\mathcal{Y}' = \{1, ..., K - 1\})$ , and unlabeled data from target domain  $T: \mathcal{X} \times (\mathcal{Y} = \{1, ..., K\})$ , the goal is to learn a target classifier  $h: \mathcal{X} \rightarrow \mathcal{V}$ .

### Remark

- · OSDA can be considered as PU with covariate shift where  $P_{S'}(x|y) \neq P_T(x|y)$  for  $y \in \mathcal{Y}'$
- · PU learning can be applied in OSDA if the gap is bridged

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# PU Learning induced Joint Error based OSDA (PUJE)

### Theorem (Approximated Joint Error based Target Upper Bound)

Given  $\mathcal{K} = \{k | k \in \mathbb{R}^K : \sum_{y \in \mathcal{Y}} k[y] = 1, k[y] \in [0,1]\},$  let  $f_S, f_T : \mathcal{X} \to \mathcal{K}$  be the true labeling functions for the source and target domains and  $\epsilon : \mathcal{K} \times \mathcal{K} \to \mathbb{R}$  denote a distance metric and  $\epsilon_D(f,f'):=\mathbb{E}_{\mathsf{x} \sim D} \, \epsilon(f(\mathsf{x}),f'(\mathsf{x}))$  measure the expected disagreement between the outputs of  $f, f': \mathcal{X} \to \mathcal{K}$ . For  $\forall f^*_\mathcal{S}, f^*_\mathcal{T}, h\in\mathcal{H}:\mathcal{X}\rightarrow\mathcal{K}$ , the expected target error is bounded by  $\epsilon_{\mathcal{T}}(h) \leq \epsilon_{\mathcal{S}}(h) + \epsilon_{\mathcal{T}}(f^*_{\mathcal{S}}, f^*_{\mathcal{T}}) + \epsilon_{\mathcal{T}}(h, f^*_{\mathcal{S}}) - |\epsilon_{\mathcal{S}}(f^*_{\mathcal{S}}, f^*_{\mathcal{T}}) - \epsilon_{\mathcal{S}}(h, f^*_{\mathcal{T}})| + \theta$  $\theta = 2\epsilon_{\tau}(f_S, f_S^*) + \epsilon_S(f_S, f_S^*) + 2\epsilon_S(f_T^*, f_T) + \epsilon_T(f_T^*, f_T) = \theta_{f_S} + \theta_{f_T}$ 

### Assumption (1)

Assume that there exists approximated labeling functions  $f^*_5, f^*_7$ such that the empirical deviations  $\hat{\theta}_{f_{\mathcal{S}}}, \hat{\theta}_{f_{\mathcal{T}}}$  measured on finite samples  $\hat{S}$ ,  $\hat{T}$  are close enough to zero.

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# Multi-class PU Learning

### Definition (Unknown Predictive Discrepancy)

Let  $v : \mathcal{K} \times \mathcal{K} \to \mathbb{R}$  denote the Unknown Predictive Discrepancy as a distance metric and  $v_D(f,f'):=\mathbb{E}_{\mathsf{x} \sim D}\, v(f(\mathsf{x}),f'(\mathsf{x}))$  measure the expected disagreement between the  $K$ -th outputs of  $f,f':\mathcal{X}\to\mathcal{K}.$  Let  $e^\mathcal{K}:\mathcal{X}\to[0,...,1]\in\mathcal{K}$  denote a function that can predict any input as unknown. The deviation from  $e^\mathcal{K}$  for a hypothesis  $h \in \mathcal{H}$  is referred to as the shorthand  $v_D(\mathit{h}) := v_D(\mathit{h},\mathit{e}^K)$  that measures the probability that samples from D have not been classified as unknown.

### Assumption (2)

Let  $S^i = P_S(x|y=i),$   $T^i = P_T(x|y=i)$  denote class conditional distributions,  $S' = P_S(x|y \neq K)$ ,  $T' = P_T(x|y \neq K)$  indicate incomplete domains that do not contain unknown class  $\mathcal{S}^{\mathsf{K}}, \mathcal{T}^{\mathsf{K}}.$ Given a feature extractor  $g: \mathcal{X} \to \mathcal{Z}$ , assume that the feature space can be aligned:  $P_{\varsigma K}(z) = P_{\tau K}(z)$ ,  $P_{\varsigma'}(z) = P_{\tau'}(z)$ .

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#### Lemma (Estimated Source Error and Discrepancy)

Let  $\sum_{i=1}^K \pi_S^i = 1, \sum_{i=1}^K \pi_T^i = 1$  denote the label distribution of S and T respectively. Given feature extractor g, approximated labeling functions can be decomposed such that  $f_{\mathcal{S}}^{*}=f_{\mathcal{S}}^{\star}\circ g, f_{\mathcal{T}}^{*}=f^{\star}\circ g$ . Given Assumption (2), the expected error on S can be estimated by the error on S' and Unknown Predictive Discrepancy on T with a mild condition that  $\pi_S^K = \pi_I^K = 1 - \alpha$ :

$$
\epsilon_S(h \circ g) = \alpha[\epsilon_{S'}(h \circ g) - v_{S'}(h \circ g)] + v_{T}(h \circ g)
$$
  

$$
\epsilon_S(f_S^* \circ g, f_T^* \circ g) = \alpha[\epsilon_{S'}(f_S^* \circ g, f_T^* \circ g) - v_{S'}(f_S^* \circ g, f_T^* \circ g)] + v_{T}(f_S^* \circ g, f_T^* \circ g)
$$
  

$$
\epsilon_S(h \circ g, f_T^* \circ g) = \alpha[\epsilon_{S'}(h \circ g, f_T^* \circ g) - v_{S'}(h \circ g, f_T^* \circ g)] + v_{T}(h \circ g, f_T^* \circ g)
$$

## Mechanism



Figure: Intuitive explanation of the difference between PUJE and existing methods. (a) existing methods do not explicitly minimize joint error and cannot group unknown class as a single cluster; (b) our proposal is an upper bound of joint error which can address large domain shift and group unknown class into a single cluster.

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### <span id="page-6-0"></span>**Results**



Figure: T-SNE visualization of feature distributions in  $(a)-(d)$ : Ar $\rightarrow$ Cl task (Office-Home dataset); (e)-(h): Syn2Real-O task.

Remark

· PUJE achieves a better alignment with a more discriminative class-wise decision boundary for unknown class, especially when the domain shift is large.

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