# Open-set Domain Adaptation via Joint Error based Multi-class Positive and Unlabeled Learning ECCV 2024

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## **Problem Setting**

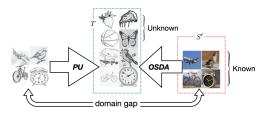


Figure: Given labeled data from source domain  $S' : \mathcal{X} \times (\mathcal{Y}' = \{1, ..., K - 1\})$ , and unlabeled data from target domain  $T : \mathcal{X} \times (\mathcal{Y} = \{1, ..., K\})$ , the goal is to learn a target classifier  $h : \mathcal{X} \to \mathcal{Y}$ .

#### Remark

- · OSDA can be considered as PU with covariate shift where  $P_{S'}(x|y) \neq P_T(x|y)$  for  $y \in \mathcal{Y}'$
- $\cdot\,$  PU learning can be applied in OSDA if the gap is bridged

## PU Learning induced Joint Error based OSDA (PUJE)

### Theorem (Approximated Joint Error based Target Upper Bound)

Given  $\mathcal{K} = \{k | k \in \mathbb{R}^{K} : \sum_{y \in \mathcal{Y}} k[y] = 1, k[y] \in [0, 1]\}$ , let  $f_{S}, f_{T} : \mathcal{X} \to \mathcal{K}$  be the true labeling functions for the source and target domains and  $\epsilon : \mathcal{K} \times \mathcal{K} \to \mathbb{R}$  denote a distance metric and  $\epsilon_{D}(f, f') := \mathbb{E}_{x \sim D} \epsilon(f(x), f'(x))$  measure the expected disagreement between the outputs of  $f, f' : \mathcal{X} \to \mathcal{K}$ . For  $\forall f_{S}^{*}, f_{T}^{*}, h \in \mathcal{H} : \mathcal{X} \to \mathcal{K}$ , the expected target error is bounded by  $\epsilon_{T}(h) \leq \epsilon_{S}(h) + \epsilon_{T}(f_{S}^{*}, f_{T}^{*}) + \epsilon_{T}(h, f_{S}^{*}) - |\epsilon_{S}(f_{S}^{*}, f_{T}^{*}) - \epsilon_{S}(h, f_{T}^{*})| + \theta$  $\theta = 2\epsilon_{T}(f_{S}, f_{S}^{*}) + \epsilon_{S}(f_{S}, f_{S}^{*}) + 2\epsilon_{S}(f_{T}^{*}, f_{T}) + \epsilon_{T}(f_{T}^{*}, f_{T}) = \theta_{f_{S}} + \theta_{f_{T}}$ 

### Assumption (1)

Assume that there exists approximated labeling functions  $f_S^*, f_T^*$  such that the empirical deviations  $\hat{\theta}_{f_S}, \hat{\theta}_{f_T}$  measured on finite samples  $\hat{S}, \hat{T}$  are close enough to zero.

## Multi-class PU Learning

### Definition (Unknown Predictive Discrepancy)

Let  $v: \mathcal{K} \times \mathcal{K} \to \mathbb{R}$  denote the Unknown Predictive Discrepancy as a distance metric and  $v_D(f, f') := \mathbb{E}_{x \sim D} v(f(x), f'(x))$  measure the expected disagreement between the K-th outputs of  $f, f': \mathcal{X} \to \mathcal{K}$ . Let  $e^{\mathcal{K}}: \mathcal{X} \to [0, ..., 1] \in \mathcal{K}$  denote a function that can predict any input as unknown. The deviation from  $e^{\mathcal{K}}$  for a hypothesis  $h \in \mathcal{H}$  is referred to as the shorthand  $v_D(h) := v_D(h, e^{\mathcal{K}})$  that measures the probability that samples from D have not been classified as unknown.

### Assumption (2)

Let  $S^{i} = P_{S}(x|y = i)$ ,  $T^{i} = P_{T}(x|y = i)$  denote class conditional distributions,  $S' = P_{S}(x|y \neq K)$ ,  $T' = P_{T}(x|y \neq K)$  indicate incomplete domains that do not contain unknown class  $S^{K}$ ,  $T^{K}$ . Given a feature extractor  $g : \mathcal{X} \to \mathcal{Z}$ , assume that the feature space can be aligned:  $P_{S^{K}}(z) = P_{T^{K}}(z)$ ,  $P_{S'}(z) = P_{T'}(z)$ .

#### Lemma (Estimated Source Error and Discrepancy)

Let  $\sum_{i=1}^{K} \pi_{S}^{i} = 1$ ,  $\sum_{i=1}^{K} \pi_{T}^{i} = 1$  denote the label distribution of S and T respectively. Given feature extractor g, approximated labeling functions can be decomposed such that  $f_{S}^{*} = f_{S}^{*} \circ g$ ,  $f_{T}^{*} = f^{*} \circ g$ . Given Assumption (2), the expected error on S can be estimated by the error on S' and Unknown Predictive Discrepancy on T with a mild condition that  $\pi_{S}^{K} = \pi_{T}^{K} = 1 - \alpha$ :

$$\begin{aligned} \epsilon_{S}(h \circ g) &= \alpha [\epsilon_{S'}(h \circ g) - v_{S'}(h \circ g)] + v_{T}(h \circ g) \\ \epsilon_{S}(f_{S}^{\star} \circ g, f_{T}^{\star} \circ g) &= \alpha [\epsilon_{S'}(f_{S}^{\star} \circ g, f_{T}^{\star} \circ g) - v_{S'}(f_{S}^{\star} \circ g, f_{T}^{\star} \circ g)] + v_{T}(f_{S}^{\star} \circ g, f_{T}^{\star} \circ g) \\ \epsilon_{S}(h \circ g, f_{T}^{\star} \circ g) &= \alpha [\epsilon_{S'}(h \circ g, f_{T}^{\star} \circ g) - v_{S'}(h \circ g, f_{T}^{\star} \circ g)] + v_{T}(h \circ g, f_{T}^{\star} \circ g) \end{aligned}$$

### Mechanism

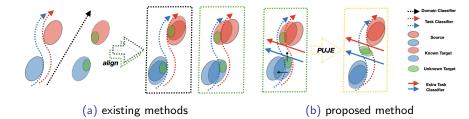


Figure: Intuitive explanation of the difference between PUJE and existing methods. (a) existing methods do not explicitly minimize joint error and cannot group unknown class as a single cluster; (b) our proposal is an upper bound of joint error which can address large domain shift and group unknown class into a single cluster.



### Results

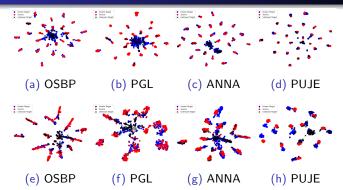


Figure: T-SNE visualization of feature distributions in (a)-(d):  $Ar \rightarrow CI$  task (Office-Home dataset); (e)-(h): Syn2Real-O task.

Remark

 PUJE achieves a better alignment with a more discriminative class-wise decision boundary for unknown class, especially when the domain shift is large.

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