



Radiative Gaussian Splatting for Efficient X-ray Novel View Synthesis

Yuanhao Cai, Yixun Liang, Jiahao Wang, Angtian Wang, Yulun Zhang, Xiaokang Yang, Zongwei Zhou, and Alan Yuille

> Johns Hopkins University, HKUST (GZ), Shanghai Jiao Tong University





• Introduction

• Method





Introduction

• Method



To reduce the harm of X-ray exposure, this work studies the low-dose X-ray reconstruction problem

- X-ray Novel View Synthesis
- Computed Tomography Reconstruction

Natural light imaging is based on the reflection off the surface of the object \rightarrow anisotropic, view dependent

X-ray imaging is based on penetration and attenuation through the object \rightarrow isotropic, view independent









Introduction

• Method







- Angle-pose Cuboid Uniform Initialization
- Radiative Gaussian Point Cloud Model
- Differentiable Radiative Rasterization



 \mathcal{P}





$$\mathbf{M}_{ext} = \begin{bmatrix} -\sin\phi & \cos\phi & 0 & 0\\ 0 & 0 & -1 & 0\\ -\cos\phi & -\sin\phi & 0 & L_{SO}\\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$\mathbf{M}_{int} = \begin{bmatrix} L_{SD} & 0 & W/2 & 0\\ 0 & L_{SD} & H/2 & 0\\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$= \{ (\frac{n_1 S_1 d}{n_1 N_1 M_2}, \frac{n_2 S_2 d}{n_2 M_2 M_2}, \frac{n_3 S_3 d}{n_1 M_2}) \mid - [\frac{M_i}{n_1}] = 1 \le n_i \le [\frac{M_i}{n_1 M_2}] \}$$

 $= \Big\{ \Big(\frac{n_1 S_1 a}{M_1}, \frac{n_2 S_2 a}{M_2}, \frac{n_3 S_3 a}{M_3} \Big) \mid - \Big[\frac{M_i}{2d}\Big] - 1 \le n_i \le \Big[\frac{M_i}{2d}\Big] + 1, \ i = 1, 2, 3 \Big\},$

Our method is SFM-free !







(a) Original RGB Gaussian Point Cloud

$$\mathbf{c}(\mathbf{d},\mathbf{k}) = \sum_{l=0}^{L} \sum_{m=-l}^{l} k_l^m Y_l^m(heta,\phi),$$

Use Spherical Harmonics – view dependent



(b) Our Radiative Gaussian Point Cloud

$$\mathcal{G}_x = \{G_i(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i, \alpha_i, \mathbf{f}_i) \mid i = 1, 2, \dots, N_p\},\$$
$$\mathbf{i}(\mathbf{f}) = \operatorname{RIRF}(\mathbf{f}) = \operatorname{Sigmoid}(\boldsymbol{\lambda} \odot \mathbf{f}),\$$

Radiation Intensity Response Function – view dependent





$$\mathbf{I} = F_{\text{DRR}}(\mathbf{M}_{ext}, \mathbf{M}_{int}, \{G_i(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i, \alpha_i, \mathbf{f}_i) \mid i = 1, 2, \dots, N_p\}),$$
$$P(\mathbf{x} | \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i) = \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_i)^\top \boldsymbol{\Sigma}_i^{-1}(\mathbf{x} - \boldsymbol{\mu}_i)\right).$$

$$\widetilde{\mathbf{t}}_i = \begin{bmatrix} \mathbf{t}_i \\ 1 \end{bmatrix} = \mathbf{M}_{ext} \ \widetilde{\boldsymbol{\mu}}_i = \mathbf{M}_{ext} \ \begin{bmatrix} \boldsymbol{\mu}_i \\ 1 \end{bmatrix}, \quad \widetilde{\mathbf{u}}_i = \begin{bmatrix} \mathbf{u}_i \\ 1 \end{bmatrix} = \mathbf{M}_{int} \ \widetilde{\mathbf{t}}_i = \mathbf{M}_{int} \ \begin{bmatrix} \mathbf{t}_i \\ 1 \end{bmatrix},$$

 $\mathbf{\Sigma}_{i}^{'} = \mathbf{J}_{i} \mathbf{W}_{i} \mathbf{\Sigma}_{i} \mathbf{W}_{i}^{ op} \mathbf{J}_{i}^{ op},$

(c) Differentiable Radiative Rasterization

$$\mathbf{J}_{i} = \begin{bmatrix} \frac{L_{SD}}{t_{z}} & 0 & -\frac{L_{SD} & t_{x}}{t_{z}^{2}} \\ 0 & \frac{L_{SD}}{t_{z}} & -\frac{L_{SD} & t_{y}}{t_{z}^{2}} \\ 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{W}_{i} = \begin{bmatrix} -\sin\phi & \cos\phi & 0 \\ 0 & 0 & -1 \\ -\cos\phi & -\sin\phi & 0 \end{bmatrix},$$

Image Coordinate System —> Camera Coordinate System —> World Coordinate System





Introduction

• Method





Method	Infer Speed	Train Time	Chest	Chest Foot Hea		Abdomen	Pancreas	Average	
InTomo [58]	$0.62 \mathrm{~fps}$	125 min	28.948 0.9915	$39.482 \\ 0.9979$	$34.832 \\ 0.9977$	$27.641 \\ 0.9646$	20.031 0.8537	30.187 0.9611	
NeRF [37]	$0.14 \mathrm{~fps}$	313 min	36.157 0.9988	41.053 0.9989	29.760 0.9991	$24.620 \\ 0.9559$	19.853 0.8560	30.289 0.9617	
TensoRF [10]	$0.77 \mathrm{~fps}$	178 min	$23.609 \\ 0.9402$	37.728 0.9929	34.429 0.9879	27.382 0.8730	29.235 0.8031	30.477 0.9194	
NeAT [42]	1.78 fps	69 min	40.765 0.9990	38.236 0.9963	27.738 0.9295	$26.741 \\ 0.8563$	$37.526 \\ 0.9017$	34.201 0.9366	
NAF [60]	2.01 fps	$63 \min$	42.366 0.9993	38.353 0.9913	30.174 0.9531	$37.590 \\ 0.9855$	$36.228 \\ 0.8844$	$36.942 \\ 0.9627$	
X-Gaussian	148 fps	9 min	43.887 0.9998	42.153 0.9997	41.579 0.9997	45.762 0.9999	43.640 0.9976	43.404 0.9993	

On the novel view synthesis task, our X-Gaussian is 6.5 dB higher and 73 x faster in model inference speed.







Our X-Gaussian can reconstruct more structural details



ECC/	

Method	+ None $+$ InTomo [58]		+ NeRF [37] + TensoRF [10]			+ NeAT [42]		+ NAF [60]		+ X-Gaussian				
Metric	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
FDK	7.41	0.093	20.31	0.498	20.57	0.502	20.61	0.501	20.94	0.511	21.28	0.523	22.60	0.584
\varDelta FDK	0	0	12.90	0.405	13.16	0.409	13.20	0.408	13.52	0.418	13.87	0.430	15.19	0.491
SART	17.24	0.528	26.28	0.859	26.78	0.853	27.06	0.867	27.31	0.869	27.84	0.879	30.25	0.907
Δ SART	0	0	9.04	0.331	9.54	0.325	9.82	0.339	10.07	0.341	10.60	0.351	13.01	0.379
ASD-POCS	17.03	0.525	25.44	0.847	26.58	0.857	26.93	0.868	26.95	0.865	27.91	0.880	30.56	0.926



On the sparse-view CT reconstruction task. Our method significantly boosts the performance of traditional methods By over 10 dB







Code, models, and data are publicly available at

https://github.com/caiyuanhao1998/SAX-NeRF